

ROMANIAN MATHEMATICAL MAGAZINE

In $\triangle ABC$ the following relationship holds:

$$\left(\frac{m_a}{h_a}\right)^{\frac{m_a}{h_a}} \cdot \left(\frac{w_a}{h_a}\right)^{\frac{w_a}{h_a}} + \frac{w_a}{h_a} \geq \frac{m_a}{h_a} + 1$$

Proposed by Daniel Sitaru – Romania

Solution by Tapas Das – India

$$\begin{aligned} & \left(\frac{m_a}{h_a}\right)^{\frac{m_a}{h_a}} \cdot \left(\frac{w_a}{h_a}\right)^{-\frac{w_a}{h_a}} = \left(\frac{m_a}{h_a}\right)^{\frac{m_a}{h_a}} \cdot \frac{1}{\left(\frac{w_a}{h_a}\right)^{\frac{w_a}{h_a}}} \\ & \geq \left(\frac{m_a}{h_a}\right)^{\frac{w_a}{h_a}} \cdot \frac{1}{\left(\frac{w_a}{h_a}\right)^{\frac{w_a}{h_a}}} \quad (\because w_a \leq m_a; w_a \geq h_a) = \left(\frac{m_a}{w_a}\right)^{\frac{w_a}{h_a}} = \left[1 + \left(\frac{m_a}{w_a} - 1\right)\right]^{\frac{w_a}{h_a}} \\ & \stackrel{\text{Bernoulli}}{\geq} 1 + \frac{w_a}{h_a} \left(\frac{m_a}{w_a} - 1\right) = 1 + \frac{m_a}{h_a} - \frac{w_a}{h_a} \\ & \therefore \left(\frac{m_a}{h_a}\right)^{\frac{m_a}{h_a}} \cdot \left(\frac{w_a}{h_a}\right)^{-\frac{w_a}{h_a}} + \frac{w_a}{h_a} \geq 1 + \frac{m_a}{h_a} - \frac{w_a}{h_a} + \frac{w_a}{h_a} = 1 + \frac{m_a}{h_a} \end{aligned}$$