ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$\frac{r_a}{h_a^2}+\frac{r_b}{h_b^2}+\frac{r_c}{h_c^2}\geq\frac{4r}{R^2}$$

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Solution by Daniel Sitaru-Romania

$$\frac{r_a}{h_a^2}+\frac{r_b}{h_b^2}+\frac{r_c}{h_c^2}=\frac{\frac{F}{s-a}}{\left(\frac{2F}{a}\right)^2}+\frac{\frac{F}{s-b}}{\left(\frac{2F}{b}\right)^2}+\frac{\frac{F}{s-c}}{\left(\frac{2F}{c}\right)^2}=$$

$$=\frac{1}{4F}\left(\frac{a^2}{s-a}+\frac{b^2}{s-b}+\frac{c^2}{s-c}\right)\overset{BERGSTROM}{\geq}$$

$$\geq\frac{1}{4F}\cdot\frac{(a+b+c)^2}{s-a+s-b+s-c}=\frac{1}{4F}\cdot\frac{4s^2}{s}=\frac{1}{4rs}\cdot\frac{4s^2}{s}=\frac{1}{r}=$$

$$=\frac{R^2}{R^2r}=\frac{R\cdot R}{R^2r}\overset{EULER}{\geq}\frac{2r\cdot 2r}{R^2r}=\frac{4r}{R^2}$$

Equality holds for $a=b=c$.