ROMANIAN MATHEMATICAL MAGAZINE

If H −orthocenter in acute ΔABC, AD, BE, CF −altitudes,

$HD = x, HE = y, HF = z$ then:

$$\frac{xy}{z}+\frac{yz}{x}+\frac{zx}{y}\ge \frac{2}{R}\cdot\left(R^2-r^2\right)$$

Proposed by Ertan Yildirim-Turkiye

Solution by Daniel Sitaru-Romania

$$cosB=\frac{BD}{AB}\Rightarrow BD=c\cdot cosB$$

$$tan\left(\frac{\pi}{2}-C\right)=\frac{HD}{BD}\Rightarrow HD=BD\cdot cosC=c\cdot cosBcotC=$$
$$=2RsinCcosB\cdot\frac{cosC}{sinC}=2RcosBcosC$$

$$\begin{cases}x=2RcosBcosC\\ y=2RcosCcosA\\ z=2RcosAcosB\end{cases}\Rightarrow\sum_{cyc}\frac{xy}{z}=\sum_{cyc}\frac{2RcosBcosC\cdot 2RcosCcosA}{2RcosAcosB}=$$

$$=2R\cdot\sum_{cyc}cos^2C=2R\cdot\frac{6R^2+4Rr+r^2-s^2}{2R^2}=$$

$$=\frac{1}{R}\left(6R^2+4Rr+r^2-s^2\right)\overset{GERRETSEN}{\underbrace{\ge}}$$

$$\ge\frac{1}{R}\left(6R^2+4Rr+r^2-4R^2-4Rr-3r^2\right)=\frac{2}{R}\cdot\left(R^2-r^2\right)$$

Equality holds for $a=b=c$.