In acute $\triangle ABC$, AD, BE, CF are altitudes and r is inradii. If r_1, r_2, r_3 are inradies of $\triangle AFE, \triangle BDF, \triangle CDE$ then:

$$r_1 + r_2 + r_3 \leq \frac{3r}{2}$$

Proposed by Ertan Yildirim-Turkiye

Solution by Daniel Sitaru-Romania

$$\sum_{cyc} r_1 = \sum_{cyc} \frac{[AFE]}{\frac{AF+FE+EA}{2}} = \sum_{cyc} \frac{\frac{1}{2} \cdot AE \cdot AF \cdot sinA}{\frac{1}{2}(acosA + bcosA + ccosA)} =$$

$$= \sum_{cyc} \frac{bcosA \cdot ccosA \cdot sinA}{(a + b + c) \cdot cosA} = \sum_{cyc} \frac{bcsinAcosA}{2s} =$$

$$= \sum_{cyc} \frac{2FcosA}{2s} = \sum_{cyc} \frac{rscosA}{s} = r\sum_{cyc} cosA \overset{JENSEN}{\lesssim}$$

$$\leq 3rcos\left(\frac{A + B + C}{3}\right) = 3rcos\frac{\pi}{3} = \frac{3r}{2}$$

Equality holds for $a = b = c$.