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In ΔABC the following relationship holds:

$$\frac{AI\cdot II_a}{b+c}+\frac{BI\cdot II_b}{c+a}+\frac{CI\cdot II_c}{a+b}\geq 2\sqrt{3}r$$

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Solution by Daniel Sitaru-Romania

$$AI=\frac{r}{\sin\frac{A}{2}}\ \ BI=\frac{r}{\sin\frac{B}{2}}\ \ CI=\frac{r}{\sin\frac{C}{2}}$$

$$II_a=4R\sin\frac{A}{2}\ \ II_b=4R\sin\frac{B}{2}\ \ II_c=4R\sin\frac{C}{2}$$

$$\frac{AI\cdot II_a}{b+c}+\frac{BI\cdot II_b}{c+a}+\frac{CI\cdot II_c}{a+b}=\sum_{cyc}\frac{AI\cdot II_a}{b+c}=\sum_{cyc}\frac{\frac{r}{\sin\frac{A}{2}}\cdot 4R\sin\frac{A}{2}}{b+c}=$$

$$=4Rr\sum_{cyc}\frac{1}{b+c}\overset{BERGSTROM}{\geq}4Rr\cdot\frac{(1+1+1)^2}{b+c+c+a+a+b}=$$

$$=4Rr\cdot\frac{9}{4s}=Rr\cdot\frac{9}{s}\overset{MITRINOVIC}{\geq}Rr\cdot\frac{9}{\frac{3\sqrt{3}}{2}R}=\frac{6r}{\sqrt{3}}=2\sqrt{3}r$$

Equality holds for $a=b=c$.