ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$\frac{b+c-a}{m_a}+\frac{a+c-b}{m_b}+\frac{a+b-c}{m_c}\le 2R\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$$

Proposed by Ertan Yildirim-Turkiye

Solution by Tapas Das-India

$$(a+b+c)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)\overset{C-S}{\ge} 9\ \ (1)$$

$$\sum\sqrt{s-a}\overset{CBS}{\le}\sqrt{3(3s-a-b-c)}=\sqrt{3s}\ \ (2)$$

$$\frac{b+c-a}{m_a}+\frac{a+c-b}{m_b}+\frac{a+b-c}{m_c}\overset{m_a\ge\sqrt{s(s-a)}}{\le}$$

$$\frac{2(s-a)}{\sqrt{s(s-a)}}+\frac{2(s-b)}{\sqrt{s(s-b)}}+\frac{2(s-c)}{\sqrt{s(s-c)}}=\frac{2}{\sqrt{s}}\sum\sqrt{s-a}\overset{(2)}{\le}\frac{2}{\sqrt{s}}\cdot\sqrt{3s}=2\sqrt{3}=$$

$$=2\cdot\frac{9}{3\sqrt{3}}\overset{(1)}{\le}\frac{2}{3\sqrt{3}}(a+b+c)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)=\frac{4s}{3\sqrt{3}}\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)\overset{Mitrinovic}{\le}$$

$$\le\frac{\frac{4(3\sqrt{3}R)}{2}}{3\sqrt{3}}\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)=2R\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$$

Equality holds for $a=b=c$.