ROMANIAN MATHEMATICAL MAGAZINE

In acute ΔABC the following relationship holds:

$$\frac{b^2+c^2-a^2}{b+c-a}+\frac{a^2+c^2-b^2}{a+c-b}+\frac{a^2+b^2-c^2}{a+b-c}\le 3R\sum_{cyc} cotA$$

Proposed by Ertan Yildirim-Turkiye

Solution by Mirsadix Muzefferov-Azerbaijan

$$\frac{4FcotA}{2(s-a)}+\frac{4FcotB}{2(s-b)}+\frac{4FcotC}{2(s-c)}=$$

$$=\frac{2FcotA}{s-a}+\frac{2FcotB}{s-b}+\frac{2FcotC}{s-c}\overset{\boxed{cotA=\frac{b^2+c^2-a^2}{4F}}(True),\ \boxed{F=(s-a).r_a}(True)}{=}$$

$$=\frac{2(s-a)r_a}{s-a}\cdot cotA+\frac{2(s-b)r_b}{s-b}\cdot cotB+\frac{2(s-c)r_c}{s-c}\cdot cotC=$$

$$=2(r_a cotA+r_b cotB+r_c cotC)$$

$$WLOG\ :\ a\le b\le c\ \rightarrow\ r_a\le r_b\le r_c\ \ and\ \ cotA\ge cotB\ge cotC$$

$$2(r_a cotA+r_b cotB+r_c cotC)\overset{CEBYSHEV}{\le}2\cdot\frac{1}{3}(r_a+r_b+r_c)\sum_{cyc} cotA=$$

$$=\frac{2}{3}(4R+r)\sum_{cyc} cotA=\frac{1}{3}(8R+2r)\sum_{cyc} cotA\overset{Euler}{\le}\frac{1}{3}\cdot 9R\sum_{cyc} cotA=3R\sum_{cyc} cotA$$

Equality holds for: $a=b=c$.