ROMANIAN MATHEMATICAL MAGAZINE

If H –orthocenter in acute $\Delta ABC, AD, BE, CF$ –altitudes, $HD = x$, $HE = y, HF = z$ then:

$$\frac{x}{bc} + \frac{y}{ca} + \frac{z}{ab} \le \frac{1}{4r}$$

Proposed by Ertan Yildirim-Izmir-Turkiye

Solution by Daniel Sitaru-Romania

$$\cos B = \frac{BD}{AB} \Rightarrow BD = c\cos B$$

$$\tan(\sphericalangle HBD) = \frac{HD}{BD} \Rightarrow HD = BD\tan\left(\frac{\pi}{2} - C\right) = c\cos B\tan\left(\frac{\pi}{2} - C\right) =$$

$$= c\cos B\cot C = c\cos B \cdot \frac{\cos C}{\sin C} = 2R\sin C\cos B \cdot \frac{\cos C}{\sin C} = 2R\cos B\cos C$$

$$\frac{x}{bc} + \frac{y}{ca} + \frac{z}{ab} = \frac{2R\cos B\cos C}{bc} + \frac{2R\cos C\cos A}{ca} + \frac{2R\cos A\cos B}{ab} =$$

$$= \frac{2R\cos B\cos C}{2R\sin B \cdot 2R\sin C} + \frac{2R\cos C\cos A}{2R\sin C \cdot 2R\sin A} + \frac{2R\cos A\cos B}{2R\sin A \cdot 2R\sin B} =$$

$$= \frac{\cot B\cot C}{2R} + \frac{\cot C\cot A}{2R} + \frac{\cot A\cot B}{2R} =$$

$$= \frac{1}{2R}\sum_{cyc}\cot B\cot C = \frac{1}{2R} \cdot 1 \overset{EULER}{\le} \frac{1}{2 \cdot 2r} = \frac{1}{4r}$$

Equality holds for: $a = b = c$.