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In ΔABC the following relationship holds:

$$\frac{1}{\mu(A)\cdot cos^2\frac{A}{2}}+\frac{1}{\mu(B)\cdot cos^2\frac{B}{2}}+\frac{1}{\mu(C)\cdot cos^2\frac{C}{2}}\geq\frac{12}{\pi}$$

Proposed by Khaled Abd Imouti-Damascus-Syria

Solution by Daniel Sitaru-Romania

$$\frac{1}{\mu(A)\cdot cos^2\frac{A}{2}}+\frac{1}{\mu(B)\cdot cos^2\frac{B}{2}}+\frac{1}{\mu(C)\cdot cos^2\frac{C}{2}}=$$

$$=\frac{\frac{1}{cos^2\frac{A}{2}}}{\mu(A)}+\frac{\frac{1}{cos^2\frac{B}{2}}}{\mu(B)}+\frac{\frac{1}{cos^2\frac{C}{2}}}{\mu(C)}\overset{BERGSTROM}{\geq}$$

$$\geq\frac{\left(\frac{1}{cos\frac{A}{2}}+\frac{1}{cos\frac{B}{2}}+\frac{1}{cos\frac{C}{2}}\right)^2}{\mu(A)+\mu(B)+\mu(C)}=\frac{1}{\pi}\left(\frac{1}{cos\frac{A}{2}}+\frac{1}{cos\frac{B}{2}}+\frac{1}{cos\frac{C}{2}}\right)^2\geq$$

$$\overset{JENSEN}{\geq}\frac{1}{\pi}\left(3\cdot\frac{1}{cos\left(\frac{A+B+C}{6}\right)}\right)^2=\frac{1}{\pi}\left(3\cdot\frac{1}{cos\left(\frac{\pi}{6}\right)}\right)^2=\frac{1}{\pi}\left(3\cdot\frac{1}{\frac{\sqrt{3}}{2}}\right)^2=\frac{12}{\pi}$$

Equality holds for $A=B=C=\frac{\pi}{3}$.