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In $\triangle ABC$ the following relationship holds:

$$\sum_{cyc} \left(\frac{\cos^2 \frac{A}{2} + 2\sin^2 \frac{A}{4}}{3} \right) > \frac{3}{4}$$

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Solution by Mirsadix Muzefferov-Azerbaijan

$$\begin{aligned} \frac{\cos^2 \frac{A}{2} + 2\sin^2 \frac{A}{4}}{3} &= \frac{1 + \cos A}{2} + \frac{(1 - \cos \frac{A}{2})}{2} = \frac{1 + \cos A + 2 - 2\cos \frac{A}{2}}{6} = \\ \frac{3 + \cos A - 2\cos \frac{A}{2}}{6} &= \frac{3 + 2\cos^2 \frac{A}{2} - 1 - 2\cos \frac{A}{2}}{6} = \frac{2 + 2\cos^2 \frac{A}{2} - 2\cos \frac{A}{2}}{6} = \\ \frac{1 + \cos^2 \frac{A}{2} - \cos \frac{A}{2} + \frac{1}{4} - \frac{1}{4}}{3} &= \frac{3}{4} + \frac{(\cos \frac{A}{2} - \frac{1}{2})^2}{3} > \frac{1}{4} \end{aligned}$$