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In ΔABC the following relationship holds:

$$\left(1+e^{\tan\frac{A}{2}}\right)\left(1+e^{\tan\frac{B}{2}}\right)\left(1+e^{\tan\frac{C}{2}}\right)\geq\left(1+e^{\frac{\sqrt{3}}{3}}\right)^{3}$$

Proposed by Khaled Abd Imouti-Syria

Solution by Tapas Das-India

$$\left(1+e^{\tan\frac{A}{2}}\right)\left(1+e^{\tan\frac{B}{2}}\right)\left(1+e^{\tan\frac{C}{2}}\right)\overset{Holder}{\geq}$$

$$\geq\left((1\cdot 1\cdot 1)^{\frac{1}{3}}+\left(e^{\sum\tan\frac{A}{2}}\right)^{\frac{1}{3}}\right)^{3}=\left(1+e^{\frac{4R+r}{3s}}\right)^{3}\geq\left(1+e^{\frac{\sqrt{3}}{3}}\right)^{3}$$

$$\left(since\ s^{2}\leq(4R+r)^{2}\frac{1}{3}\right)-Doucet's\ inequality$$

Equality holds for $a=b=c$.