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In ΔABC the following relationship holds:

$$\sqrt[3]{\frac{\tan^7\frac{A}{2}}{\sin^4 A}}+\sqrt[3]{\frac{\tan^7\frac{A}{2}}{\sin^4 A}}+\sqrt[3]{\frac{\tan^7\frac{A}{2}}{\sin^4 A}}\ge\frac{\sqrt[3]{16\sqrt{3}}}{3}$$

Proposed by Khaled Abd Imouti-Syria

Solution by Tapas Das-India

$$\sum\tan\frac{A}{2}=\frac{4R+r}{s}\overset{Doucet}{\ge}\sqrt{3}\ (1)\ and\ \sum\sin A=\frac{s}{R}\overset{Mitrinovic}{\le}\frac{3\sqrt{3}}{2}\ (2)$$

$$\sqrt[3]{\frac{\tan^7\frac{A}{2}}{\sin^4 A}}+\sqrt[3]{\frac{\tan^7\frac{A}{2}}{\sin^4 A}}+\sqrt[3]{\frac{\tan^7\frac{A}{2}}{\sin^4 A}}=\sum\sqrt[3]{\frac{\tan^7\frac{A}{2}}{\sin^4 A}}=\sum\frac{\left(\tan\frac{A}{2}\right)^{\frac{7}{3}}}{(\sin A)^{\frac{4}{3}}}\overset{Radon}{\ge}$$

$$\ge\frac{\left(\sum\tan\frac{A}{2}\right)^{\frac{7}{3}}}{(\sum\sin A)^{\frac{4}{3}}}\overset{(1)\&(2)}{\ge}\frac{(\sqrt{3})^{\frac{7}{3}}}{\left(\frac{3\sqrt{3}}{2}\right)^{\frac{4}{3}}}=\left(\frac{27\sqrt{3}.16}{729}\right)^{\frac{1}{3}}=\frac{\sqrt[3]{16\sqrt{3}}}{3}$$

Equality for $A=B=C$.