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In $\triangle ABC$ the following relationship holds:

$$\frac{\sin^2 A}{bc} + \frac{\sin^2 B}{ca} + \frac{\sin^2 C}{ab} \geq \frac{s^2}{R^2} \cdot \frac{1}{a^2 + b^2 + c^2}$$

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Solution by Daniel Sitaru-Romania

$$\begin{aligned} \frac{\sin^2 A}{bc} + \frac{\sin^2 B}{ca} + \frac{\sin^2 C}{ab} &= \frac{a^2}{4R^2} + \frac{b^2}{4R^2} + \frac{c^2}{4R^2} = \\ &= \frac{1}{4R^2} \cdot \left(\frac{a^2}{bc} + \frac{b^2}{ca} + \frac{c^2}{ab} \right) \stackrel{\text{BERGSTROM}}{\geq} \frac{1}{4R^2} \cdot \frac{(a+b+c)^2}{ab+bc+ca} = \\ &= \frac{4s^2}{4R^2} \cdot \frac{1}{ab+bc+ca} \geq \frac{s^2}{R^2} \cdot \frac{1}{a^2+b^2+c^2} \end{aligned}$$