ROMANIAN MATHEMATICAL MAGAZINE

In any Δ ABC with F → area, the following relationship holds :

$$F \le \frac{\sqrt{3}}{4}\max\left(\frac{a^{n+2}+b^{n+2}}{a^n+b^n},\frac{b^{n+2}+c^{n+2}}{b^n+c^n},\frac{c^{n+2}+a^{n+2}}{c^n+a^n}\right), n \in \mathbb{N}^*$$

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$$3\max\left(\frac{a^{n+2}+b^{n+2}}{a^n+b^n},\frac{b^{n+2}+c^{n+2}}{b^n+c^n},\frac{c^{n+2}+a^{n+2}}{c^n+a^n}\right) \ge \sum_{cyc}\frac{b^{n+2}+c^{n+2}}{b^n+c^n}$$

$$= \sum_{cyc}\frac{b^n.b^2+c^n.c^2}{b^n+c^n} \overset{\text{Chebyshev}}{\ge} \sum_{cyc}\frac{\frac{1}{2}(b^n+c^n)(b^2+c^2)}{b^n+c^n}$$

$$(\because b \ge c \Rightarrow b^n \ge c^n \text{ and } b^2 \ge c^2 \text{ and } b \le c \Rightarrow b^n \le c^n \text{ and } b^2 \le c^2)$$

$$= \sum_{cyc} a^2 \overset{\text{Ionescu-Weitzenbock}}{\ge} 4\sqrt{3}F$$

$$\Rightarrow \frac{\sqrt{3}}{4}\max\left(\frac{a^{n+2}+b^{n+2}}{a^n+b^n},\frac{b^{n+2}+c^{n+2}}{b^n+c^n},\frac{c^{n+2}+a^{n+2}}{c^n+a^n}\right) \ge F$$

$$\therefore F \le \frac{\sqrt{3}}{4}\max\left(\frac{a^{n+2}+b^{n+2}}{a^n+b^n},\frac{b^{n+2}+c^{n+2}}{b^n+c^n},\frac{c^{n+2}+a^{n+2}}{c^n+a^n}\right) \forall \Delta \text{ ABC and}$$

$$\forall n \in \mathbb{N}^*, '' ='' \text{ iff } \Delta \text{ ABC is equilateral (QED)}$$