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In $\Delta ABC$ the following relationship holds:

$$\sum\sqrt{\frac{r}{r_a(s^2+r_a^2)}}\le\frac{3}{2s}$$

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$$s^2+r_a^2=r_ar_b+r_br_c+r_cr_a+r_a^2=(r_a+r_b)(r_a+r_c)\ (1)$$

$$\sum\frac{1}{s^2+r_a^2}\overset{(1)}{=}\sum\frac{1}{(r_a+r_b)(r_a+r_c)}=\frac{\sum(r_a+r_b)}{(r_a+r_b)(r_a+r_c)(r_b+r_c)}=$$

$$=\frac{2\sum r_a}{(\sum r_a)(\sum r_ar_b)-r_ar_br_c}=\frac{2(4R+r)}{(4R+r)s^2-s^2r}=\frac{2(4R+r)}{4Rs^2}=$$

$$=\frac{2}{4s^2}\left(4+\frac{r}{R}\right)\overset{Euler}{\le}\frac{2}{4s^2}\left(4+\frac{1}{2}\right)=\frac{9}{4s^2}\ (2)$$

$$\sum\sqrt{\frac{r}{r_a(s^2+r_a^2)}}=\sqrt{r}\sum\sqrt{\frac{1}{r_a}}\sqrt{\frac{1}{s^2+r_a^2}}\overset{CBS}{\le}$$

$$\le\sqrt{r}\sqrt{\left(\sum\frac{1}{r_a}\right)\left(\sum\frac{1}{s^2+r_a^2}\right)}\overset{(2)}{\le}\sqrt{r}\sqrt{\frac{1}{r}\frac{9}{4s^2}}=\frac{3}{2s}$$

$$Equality\ holds\ for\ a=b=c$$