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In ΔABC the following relationship holds:

$$\sum \sqrt{\frac{4r}{r_a}+1} \leq \sqrt{21}$$

Proposed by Marin Chirciu-Romania

Solution by Tapas Das-India

$$\sum \sqrt{\frac{4r}{r_a}+1} = \sum \sqrt{\frac{4r}{\frac{rs}{s-a}}+1} = \sum \sqrt{4\frac{s-a}{s}+1} \overset{CBS}{\leq}$$

$$\leq \sqrt{3\sum\left(4\frac{s-a}{s}+1\right)} = \sqrt{3\left(\frac{4s}{s}+3\right)} = \sqrt{21}$$

Equality holds for an equilateral triangle