ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$\sum \sqrt{\frac{4r}{h_a}+1} \le \sqrt{21}$$

Proposed by Marin Chirciu-Romania

Solution by Tapas Das-India

$$\sum \sqrt{4 \cdot \frac{r}{h_a}+1} = \sum \sqrt{4 \cdot \frac{r}{\frac{2rs}{a}}+1} = \sum \sqrt{\frac{2a}{s}+1} \overset{CBS}{\le} \sqrt{3\left(\sum\left(\frac{2a}{s}+1\right)\right)} =$$

$$= \sqrt{3\left(\frac{2(a+b+c)}{s}+3\right)} = \sqrt{3\left(\frac{4s}{s}+3\right)} = \sqrt{21}$$

Equality holds for an equilateral triangle