ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$3\sqrt{3}\cdot\frac{2r}{R}\le\sum\left(\frac{1}{b}+\frac{1}{c}\right)h_a\le 3\sqrt{3}$$

Proposed by Marin Chirciu-Romania

Solution by Tapas Das-India

$$\sum\left(\frac{1}{b}+\frac{1}{c}\right)h_a=\sum\left(\frac{1}{b}+\frac{1}{c}\right)\frac{bc}{2R}=\frac{1}{2R}\sum(b+c)=\frac{2(a+b+c)}{2R}=$$

$$=\frac{2s}{R}\overset{Mitrinovic}{\le}2\cdot\frac{3\sqrt{3}R}{2R}=3\sqrt{3}$$

$$\sum\left(\frac{1}{b}+\frac{1}{c}\right)h_a=\sum\left(\frac{1}{b}+\frac{1}{c}\right)\frac{bc}{2R}=\frac{1}{2R}\sum(b+c)=\frac{2(a+b+c)}{2R}=$$

$$=\frac{2s}{R}\overset{Mitrinovic}{\ge}2\cdot\frac{3\sqrt{3}r}{R}=3\sqrt{3}\frac{2r}{R}$$

Equality holds for an equilateral triangle