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**In $\Delta ABC$ the following relationship holds:**

$$\sum w_a(h_b+h_c)\le 6\sqrt{3}F$$

***Proposed by Marin Chirciu-Romania***

***Solution by Tapas Das-India***

$$h_b+h_c=\frac{ac}{2R}+\frac{ab}{2R}=\frac{a(b+c)}{2R},\ w_a=\frac{2bc}{b+c}\cos\frac{A}{2},\ w_a(h_b+h_c)=\frac{abc}{R}\cos\frac{A}{2}$$

$$\sum\cos\frac{A}{2}=\left(\sqrt{\sum\cos\frac{A}{2}}\right)^2\overset{CBS}{\le}\sqrt{3\sum\cos^2\frac{A}{2}}=\sqrt{3\left(2+\frac{r}{2R}\right)}\overset{Euler}{\le}\sqrt{3\left(2+\frac{1}{4}\right)}=\frac{3\sqrt{3}}{2}$$

$$\sum w_a(h_b+h_c)=\sum\frac{abc}{R}\cos\frac{A}{2}=4F\sum\cos\frac{A}{2}\le 4F\frac{3\sqrt{3}}{2}=6\sqrt{3}F$$

**Equality holds for an equilateral triangle.**