ROMANIAN MATHEMATICAL MAGAZINE

In acute ΔABC the following relationship holds:

$$\frac{1}{\prod \cos A}+128\prod \sin\frac{A}{2}\ge 24$$

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Solution by Tapas Das-India

$$\frac{1}{\prod \cos A}+128\prod \sin\frac{A}{2}=\frac{4R^2}{s^2-(2R+r)^2}+128\frac{r}{4R}\overset{Gerretsen}{\ge}$$

$$\ge \frac{4R^2}{4R^2+4Rr+3r^2-(2R+r)^2}+128\frac{r}{4R}=\frac{4R^2}{2r^2}+32\frac{r}{R}=\frac{2R^2}{r^2}+\frac{32r}{R}$$

$$\boldsymbol{We\ need\ to\ show}:$$

$$\frac{2R^2}{r^2}+\frac{32r}{R}\ge 24 \ or\ 2x^2+\frac{16}{x}\overset{\frac{R}{r}=x\ge 2}{\ge} 12\ or$$

$$x^3-12x+16\ge 0\ or\ (x-2)^2(x+4)\ge 0\ true$$

Equality holds for an equilateral triangle.