ROMANIAN MATHEMATICAL MAGAZINE

In $\triangle ABC$ the following relationship holds:

$$12r \le \sum \frac{bc}{h_a} \le 6R$$

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Solution by Mirsadix Muzefferov-Azerbaijan

$$12r \le \sum \frac{bc}{h_a} \le 6R$$

$$\frac{bc}{h_a} = \frac{\frac{2S}{\sin(A)}}{\frac{2S}{a}} = \frac{a}{\sin(A)} = 2R$$

$$\sum_{cyc} \frac{bc}{h_a} = 6R$$

$$\sum_{cyc} \frac{bc}{h_a} = 6R \ge 12r \ (Proved)$$

Equality holds triangle is an equilateral.