In acute triangle ABC holds:

$$\sum_{cyc} \tan B \tan C \geq 3 + \sum_{cyc} \sqrt{1 + \tan^2 A}$$

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Let $x := \tan A$, $y := \tan B$, $z := \tan C$. We have $x, y, z > 0$ and $x + y + z = xyz$.
By $AM - GM$ inequality we have

$$3 + \sum_{cyc} \sqrt{1 + x^2} = 3 + \sum_{cyc} \sqrt{1 + \frac{x(x + y + z)}{yz}} =$$

$$= 3 + \sum_{cyc} \sqrt{\frac{(x + y)(x + z)}{yz}} \leq 3 + \sum_{cyc} \frac{y + z}{x} = \sum_{cyc} x \cdot \sum_{cyc} \frac{1}{x} = \sum_{cyc} yz,$$

as desired. Equality holds iff ΔABC is equilateral.