ROMANIAN MATHEMATICAL MAGAZINE

If $x,y,z>0$ then in ΔABC holds:

$$\sum\frac{y+z}{x}\,s_a^2\ge\left(12\sqrt{3}\right)r\frac{F}{R}$$

Proposed by Marin Chirciu-Romania

Solution by Tapas Das-India

$$LHS\overset{AM-GM}{\ge}\sum_{cyc}\frac{2\sqrt{yz}}{x}s_a^2\overset{AG-GM}{\ge}6\sqrt[3]{(s_a\,s_b\,s_c)^2}\overset{s_a\ge h_a}{\ge}6\sqrt[3]{(h_a\,h_b\,h_c)^2}=$$

$$=6\sqrt[3]{\left(\frac{4F^4}{R^2}\right)}=6F\sqrt[3]{\left(\frac{4Rrs}{R^3}\right)}\underset{Mitrinovic}{\overset{Euler}{\ge}}=12F\sqrt{3}\frac{r}{R}$$