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In ΔABC the following relationship holds:

$$\frac{m_a+m_b+m_c}{R^2}\le\frac{1}{r}\sum\cos\frac{B}{2}\cos\frac{C}{2}$$

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Solution by Tapas Das-India

$$\frac{m_a+m_b+m_c}{R^2}\overset{Leunberger}{\le}\frac{4R+r}{R^2}\overset{Euler}{\le}\frac{9R}{2R^2}=\frac{9}{2R}\quad(1)$$

$$\frac{1}{r}\sum\cos\frac{B}{2}\cos\frac{C}{2}\overset{AM-GM}{\ge}\frac{3}{r}\sqrt[3]{\prod\cos^2\frac{A}{2}}=\frac{3}{r}\sqrt[3]{\left(\frac{s^2}{16R^2}\right)}\ge$$

$$\ge\frac{3}{r}\sqrt[3]{\left(\frac{s^3}{16R^2s}\right)}\overset{Mitrinovic}{\ge}\frac{3}{r}\sqrt[3]{\frac{s^3}{8R^3 3\sqrt{3}}}=\sqrt{3}\frac{s}{2Rr}\overset{Mitrinovic}{\ge}\frac{9}{2R}\quad(2)$$

$$from(1)\ \&\ (2) we\ get\ In\ \Delta\ ABC:\frac{m_a+m_b+m_c}{R^2}\le\frac{1}{r}\sum\cos\frac{B}{2}\cos\frac{C}{2}$$