ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$\sum_{cyc} \frac{1}{3\tan^2\frac{A}{2}+2} \le 1$$

Proposed by Marin Chirciu-Romania

Solution by Tapas Das-India

$$\sum \frac{1}{3\tan^2\frac{A}{2}+2} = \frac{1}{2}\sum\left(1-\frac{3\tan^2\frac{A}{2}}{3\tan^2\frac{A}{2}+2}\right) = \frac{3}{2}-\frac{1}{2}\sum\left(\frac{3\tan^2\frac{A}{2}}{3\tan^2\frac{A}{2}+2}\right) \overset{CBS}{\le}$$

$$\le \frac{3}{2}-\frac{1}{2}\cdot\frac{3\left(\sum\tan\frac{A}{2}\right)^2}{3\sum\tan^2\frac{A}{2}+6} = \frac{1}{2}\left(3-\frac{3\left(\frac{4R+r}{s}\right)^2}{3\left(\frac{(4R+r)}{s}\right)^2-6+6}\right) = \frac{1}{2}(3-1) = 1$$

$$Eequality\ for\ \Delta ABC\ equilateral.$$