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In ΔABC the following relationship holds:

$$10-\frac{2r}{R}\le\sum a\sum\frac{1}{a}\le\frac{9R}{2r}$$

Proposed by Marin Chirciu-Romania

Solution by Tapas Das-India

$$\sum a\sum\frac{1}{a}=2s\frac{s^2+r^2+4Rr}{4Rrs}=\frac{s^2+r^2+4Rr}{2Rr}$$

$$\sum a\sum\frac{1}{a}=\frac{s^2+r^2+4Rr}{2Rr}\overset{Gerretsen}{\le}\frac{4R^2+8Rr+4r^2}{2Rr}=$$

$$=\frac{4(R+r)^2}{2Rr}\overset{Euler}{\le}\frac{4\left(R+\frac{R}{2}\right)^2}{2Rr}=\frac{9R}{2r}$$

$$\sum a\sum\frac{1}{a}=\frac{s^2+r^2+4Rr}{2Rr}\overset{Gerretsen}{\ge}\frac{20Rr-4r^2}{2Rr}=10-\frac{4r}{2R}=10-\frac{2r}{R}$$

Equality holds for $a=b=c$.