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In ΔABC the following relationship holds:

$$\sum \frac{\cos^n(B-C)}{h_a^n} \geq 3\left(\frac{2}{3R}\right)^n, n \in N$$

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Solution by Tapas Das-India

$$a\cos(B-C) = 2R\sin A\cos(B-C) = 2R\sin\{\pi-(B+C)\}\cos(B-C)$$
$$= 2R\sin(B+C)\cos(B-C) = R(\sin 2B + \sin 2C) \;\; (1)$$

$$\sum \frac{\cos^n(B-C)}{h_a^n} = \sum \frac{(a\cos(B-C))^n}{(2F)^n} \overset{CBS}{\geq} \frac{1}{(2F)^n}\cdot\frac{1}{3^{n-1}}\left(\sum a\cos(B-C)\right)^n \overset{(1)}{=}$$

$$= \frac{1}{(2F)^n}\cdot\frac{1}{3^{n-1}}\left(R(\sin 2B + \sin 2C)\right)^n = \frac{1}{(2F)^n}\cdot\frac{2^nR^n}{3^{n-1}}\left(\sum \sin 2A\right)^n =$$

$$= \frac{1}{(2F)^n}\cdot\frac{2^nR^n}{3^{n-1}}\left(4\prod \sin A\right)^n = \frac{1}{(2F)^n}\cdot\frac{2^nR^n}{3^{n-1}}.2^{2n}\left(\frac{sr}{2R^2}\right)^n = \frac{2^n}{3^{n-1}}\cdot\frac{1}{R^n} = 3\left(\frac{2}{3R}\right)^n$$

Equality holds for $a = b = c$.