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In ΔABC the following relationship holds:

$$\frac{b}{s-b}+\frac{c}{s-c}\geq\frac{8R}{r}\cdot\frac{s-a}{a}\cdot\sin\frac{A}{2}$$

Proposed by Marin Chirciu-Romania

Solution by Tapas Das-India

$$\frac{\frac{b}{s-b}+\frac{c}{s-c}}{\frac{s-a}{a}\sin\frac{A}{2}}=\frac{1}{\sin\frac{A}{2}}\left(\frac{ab}{(s-b)(s-a)}+\frac{ac}{(s-c)(s-a)}\right)=$$

$$=\frac{1}{\sin\frac{A}{2}}\left(\frac{1}{\sin^2\frac{C}{2}}+\frac{1}{\sin^2\frac{B}{2}}\right)\overset{AM-GM}{\geq}\frac{2}{\sin\frac{A}{2}}\left(\frac{1}{\sin\frac{B}{2}}\frac{1}{\sin\frac{C}{2}}\right)=2.\left(\frac{4R}{r}\right)=\frac{8R}{r}$$

$$or\ \frac{b}{s-b}+\frac{c}{s-c}\geq\frac{8R}{r}\cdot\frac{s-a}{a}\cdot\sin\frac{A}{2}$$

$$Equality\ for\ a=b=c.$$