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In $\triangle ABC$ the following relationship holds:

$$\sum \frac{h_a}{b+c} \sin A \geq \frac{9r}{4R}$$

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Solution by Tapas Das-India

$$\begin{aligned} \sum \frac{h_a}{b+c} \sin A &= \sum \frac{bc}{2R} \cdot \frac{a}{2R} \cdot \frac{1}{b+c} = \frac{abc}{4R^2} \sum \frac{1}{b+c} \stackrel{\text{Bergstrom}}{\geq} \\ &\geq \frac{abc}{4R^2} \cdot \frac{(1+1+1)^2}{2a+2b+2c} = \frac{abc}{4R^2} \cdot \frac{9}{4s} = 4Rrs \cdot \frac{9}{16R^2s} = \frac{9r}{4R} \end{aligned}$$

Equality holds for $a = b = c$