ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$\sum \frac{(a+b)^{2n}}{w_a + r_b} \ge \frac{\left(4\sqrt{3}r\right)^{2n}}{R}, n \in N$$

Proposed by Marin Chirciu-Romania

Solution by Tapas Das-India

$$\sum \frac{(a+b)^{2n}}{w_a + r_b} \overset{Holder}{\ge} \frac{(a+b+b+c+c+a)^{2n}}{\left((\sum w_a) + (\sum r_a)\right) \cdot 3^{2n-2}} \overset{w_a \le m_a}{\ge}$$

$$\ge \frac{(4s)^{2n}}{3^{2n-2} \cdot \left((\sum m_a) + (\sum r_a)\right)} \overset{Leuenberger}{\ge} \frac{(4s)^{2n}}{3^{2n-2}\left(2(4R+r)\right)} \overset{Euler\ \&\ Mitrinovic}{\ge}$$

$$\ge \frac{\left(4.3\sqrt{3}r\right)^{2n}}{3^{2n-2}\,.\,2\,.\,\frac{9R}{2}} = \frac{\left(4\sqrt{3}r\right)^{2n}}{R}$$

Equality holds for $a = b = c$