ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$\sum \frac{(a+b)^4}{w_a + r_b} \geq \frac{\left(4\sqrt{3}r\right)^4}{R}$$

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Solution by Tapas Das-India

$$\sum \frac{(a+b)^4}{w_a + r_b} \overset{Holder}{\geq} \frac{(a+b+b+c+c+a)^4}{(\sum w_a) + (\sum r_a)9} \overset{w_a \leq m_a}{\geq} \frac{256s^4}{9(\sum m_a) + (\sum r_a)} \overset{Leuenberger}{\geq}$$

$$\geq \frac{(4s)^4}{9\left(2(4R+r)\right)} \overset{Euler\ \&\ Mitrinovic}{\geq} \frac{\left(4.3\sqrt{3}r\right)^4}{9\,.\,2\,.\,\frac{9R}{2}} = \frac{81\left(4\sqrt{3}r\right)^4}{81R} = \frac{\left(4\sqrt{3}r\right)^4}{R}$$

$$Equality\ holds\ \ for\ a = b = c$$