ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$\sum \frac{r_a^2}{r_a^2+3rr_a+9r^2} \geq 1$$

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Solution by Tapas Das-India

$$\frac{r_a^2}{r_a^2+3rr_a+9r^2} = 1-\frac{3rr_a+9r^2}{r_a^2+3rr_a+9r^2} = 1-\frac{3rr_a+9r^2}{(r_a^2+9r^2)+3rr_a} \overset{AM-GM}{\geq}$$

$$\geq\ 1-\frac{3r(r_a+3r)}{6rr_a+3rr_a} = 1-\frac{r_a+3r}{3r_a} = 1-\frac{1}{3}-\frac{r}{r_a} = \frac{2}{3}-\frac{r}{r_a}\ (1)$$

$$\sum \frac{1}{r_a} = \frac{1}{F}\sum s-a\ = \frac{s}{F} = \frac{1}{r}\ (2)$$

$$\sum \frac{r_a^2}{r_a^2+3rr_a+9r^2} \overset{(1)}{\geq} \sum\left(\frac{2}{3}-\frac{r}{r_a}\right) = \frac{6}{3}-r\sum\frac{1}{r_a} \overset{(2)}{=} 2-1=1$$

Equality hols if ΔABC is equilateral.