# ROMANIAN MATHEMATICAL MAGAZINE

In $\Delta ABC$ the following relationship holds:

$$\sum \frac{h_a^2}{h_a^2+3rh_a+9r^2} \geq 1$$

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$$\sum \frac{h_a^2}{h_a^2+3rh_a+9r^2} = 1-\frac{3rh_a+9r^2}{h_a^2+3rh_a+9r^2} = 1-\frac{3rh_a+9r^2}{(h_a^2+9r^2)+3rh_a} \overset{AM-GM}{\geq}$$

$$\geq 1-\frac{3r(h_a+3r)}{6rh_a+3rh_a} = 1-\frac{h_a+3r}{3h_a} = 1-\frac{1}{3}-\frac{r}{h_a} = \frac{2}{3}-\frac{r}{h_a} \quad (1)$$

$$\sum \frac{1}{h_a} = \frac{1}{2F}\sum a = \frac{2s}{2F} = \frac{1}{r} \quad (2)$$

$$\sum\sum \frac{h_a^2}{h_a^2+3rh_a+9r^2} \overset{(1)}{\geq} \sum\left(\frac{2}{3}-\frac{r}{h_a}\right) = \frac{6}{3}-r\sum \frac{1}{h_a} \overset{(2)}{=} 2-1=1$$

*Equality holds if $\Delta ABC$ is equilateral.*