ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$12rs \leq \sum a(r_b + r_c) \leq 6Rs$$

Proposed by Marin Chirciu-Romania

Solution by Tapas Das-India

$$WLOG\ a \geq b \geq c\ \ then\ r_a \geq r_b \geq r_c$$

$$\sum a(r_b + r_c) \overset{Chebyshev}{\leq} \frac{1}{3}\left(\sum a\right)\left(\sum (r_b + r_c)\right) = \frac{1}{3} 2s.2\sum r_a =$$

$$= \frac{4}{3} s(4R + r) \overset{Euler}{\leq} \frac{4}{3} s.\left(4R + \frac{R}{2}\right) = \frac{4}{3}\frac{.9Rs}{2} = 6Rs$$

$$\sum a(r_b + r_c) \overset{AM-GM}{\geq} 2\sum a\sqrt{r_b r_c} \overset{Am-Gm}{\geq} 6\sqrt[3]{abcr_a r_b r_c} =$$

$$= 6\sqrt[3]{4Rrs.s^2 r} \overset{Euler}{\geq} 6\sqrt[3]{4.2r.rs.s^2 r} = 12rs$$

$$Equality\ holds\ for\ \ a = b = c$$