ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$\sum \frac{a^2}{(\sin B + \sin C)^2} \geq 3R^2$$

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Solution by Mirsadix Muzefferov-Azerbaijan

$$\frac{a^2}{(\sin B + \sin C)^2} = \frac{4R^2 \sin^2 A}{(\sin B + \sin C)^2} = \left(\frac{2\sin\frac{A}{2}\cos\frac{A}{2}}{2\sin\frac{B+C}{2}.\cos\frac{B-C}{2}}\right)^2.4R^2 =$$

$$4R^2\left(\frac{\sin\frac{A}{2}\cos\frac{A}{2}}{\cos\frac{A}{2}.\cos\frac{B-C}{2}}\right)^2 \geq 4R^2\sin^2\frac{A}{2}\ \left(Because\ \cos\frac{B-C}{2} \leq 1\right)$$

$$\sum_{cyc} \frac{a^2}{(\sin B + \sin C)^2} \geq 4R^2 \sum_{cyc} \sin^2\frac{A}{2}$$

$$\sum_{cyc} \sin^2\frac{A}{2} = \frac{3}{2} - \frac{1}{2}\sum_{cyc} \cos A = \frac{3}{2} - \frac{1}{2}\left(1 + \frac{r}{R}\right) = 1 - \frac{r}{2R} \overset{Euler}{\geq} 1 - \frac{1}{4} = \frac{3}{4} \quad (1)$$

$$\sum_{cyc} \frac{a^2}{(\sin B + \sin C)^2} \geq 4R^2 \cdot \frac{3}{4} = 3R^2$$