

# ROMANIAN MATHEMATICAL MAGAZINE

In  $\triangle ABC$  holds:

$$\sum_{cyc} \frac{r_a(2r_a - 3r)}{r_a^2 + 18r^2} \geq 1$$

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$$\begin{aligned} \sum_{cyc} \frac{r_a(2r_a - 3r)}{r_a^2 + 18r^2} &= \sum_{cyc} \left( 2 - \frac{3r(r_a + 12r)}{(r_a^2 + 9r^2) + 9r^2} \right) \stackrel{AM-GM}{\geq} \sum_{cyc} \left( 2 - \frac{3r(r_a + 12r)}{6rr_a + 9r^2} \right) = \\ &= \sum_{cyc} \left( 2 - \frac{r_a + 12r}{2r_a + 3r} \right) = \sum_{cyc} \left( \frac{3}{2} - \frac{21r}{2(2r_a + 3r)} \right) \stackrel{CBS}{\geq} \sum_{cyc} \left[ \frac{3}{2} - \frac{21r}{2 \cdot 9} \left( \frac{2}{r_a} + \frac{1}{3r} \right) \right] = \\ &= \sum_{cyc} \left( \frac{10}{9} - \frac{7r}{3r_a} \right) = \frac{10}{3} - \frac{7r}{3} \cdot \sum_{cyc} \frac{1}{r_a} = \frac{10}{3} - \frac{7r}{3} \cdot \frac{1}{r} = 1. \end{aligned}$$

Equality holds iff  $\triangle ABC$  is equilateral.