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In ΔABC the following relationship holds:

$$\sum\left(a^{8n}+\frac{1}{a^{4n}}\right)\geq 6\left(\frac{4F}{\sqrt{3}}\right)^{n},n\in N$$

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Solution by Tapas Das-India

$$\sum\left(a^{8n}+\frac{1}{a^{4n}}\right)\overset{AM-GM}{\geq}\sum 2\sqrt{a^{8n}\cdot\frac{1}{a^{4n}}}=2\sum a^{2n}\overset{AM-GM}{\geq}$$

$$\geq 6\sqrt[3]{((abc)^{2})^{n}}\overset{Carlitz}{\geq}6\left(\left(\frac{4F}{\sqrt{3}}\right)^{3}\right)^{\frac{n}{3}}=6\left(\frac{4F}{\sqrt{3}}\right)^{n}$$

$$Equality\ holds\ \ for\ a=b=c$$