In ΔABC the following relationship holds:

$$\sum a \sqrt{\frac{b^2 + c^2}{2}} \leq \frac{9R^4}{4r^2}$$

Proposed by Marin Chirciu-Romania

Solution by Tapas Das-India

$$\sum a \sqrt{\frac{b^2 + c^2}{2}} \overset{CBS}{\leq} \sqrt{\left(\sum a^2\right)\left(\sum \frac{b^2 + c^2}{2}\right)} = \left(\sum a^2\right) \overset{Leibniz}{\leq}$$

$$\leq 9R^2 = \frac{9R^4}{R^2} \overset{Euler}{\leq} \frac{9R^4}{(2r)^2} = \frac{9R^4}{4r^2}$$

Equality holds for $a = b = c$