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In ΔABC the following relationship holds:

$$\sum \frac{\tan\frac{A}{2}+\tan\frac{B}{2}}{\sqrt{\tan\frac{A}{2}\tan\frac{B}{2}-\tan^2\frac{A}{2}\tan^2\frac{B}{2}}} \le \frac{p\sqrt{2}}{r}$$

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Solution by Mirsadix Muzefferov-Azerbaijan

$$\frac{\tan\frac{A}{2}+\tan\frac{B}{2}}{\sqrt{\tan\frac{A}{2}\tan\frac{B}{2}-\tan^2\frac{A}{2}\tan^2\frac{B}{2}}} = \left(\frac{\tan\frac{A}{2}+\tan\frac{B}{2}}{\tan\frac{A}{2}\cdot\tan\frac{B}{2}}\right)^{\frac{1}{2}}\cdot\left(\frac{\tan\frac{A}{2}+\tan\frac{B}{2}}{1-\tan\frac{A}{2}\cdot\tan\frac{B}{2}}\right)^{\frac{1}{2}} =$$

$$= \left(\frac{\frac{\cos\frac{C}{2}}{\cos\frac{A}{2}\cdot\cos\frac{B}{2}}}{\tan\frac{A}{2}\cdot\tan\frac{B}{2}}\right)^{\frac{1}{2}}\cdot\left(\tan\frac{A}{2}+\tan\frac{B}{2}\right)^{\frac{1}{2}} = \left(\frac{\cos\frac{C}{2}}{\sin\frac{A}{2}\cdot\sin\frac{B}{2}}\right)^{\frac{1}{2}}\cdot\left(ctg\frac{C}{2}\right)^{\frac{1}{2}} =$$

$$= \frac{\cos\frac{C}{2}}{\left(\sin\frac{A}{2}\cdot\sin\frac{B}{2}\cdot\sin\frac{C}{2}\right)^{\frac{1}{2}}} = \frac{\cos\frac{C}{2}}{\left(\frac{r}{4R}\right)^{\frac{1}{2}}} = \left(\frac{4R}{r}\right)^{\frac{1}{2}}\cdot\cos\frac{C}{2} = \frac{\sqrt{4Rr}}{r}\cdot\cos\frac{C}{2} =$$

$$= \frac{1}{r}\cdot\sqrt{4Rr}\cdot\cos\frac{C}{2} = \frac{1}{r}\sqrt{\frac{abc}{p}}\cdot\sqrt{\frac{p(p-c)}{ab}} = \frac{1}{r}\sqrt{c(p-c)}$$

$$\sum \frac{\tan\frac{A}{2}+\tan\frac{B}{2}}{\sqrt{\tan\frac{A}{2}\tan\frac{B}{2}-\tan^2\frac{A}{2}\tan^2\frac{B}{2}}} = \frac{1}{r}\cdot\sum\sqrt{c(p-c)} \overset{CBS}{\le}$$

$$\le \frac{1}{r}\cdot\sqrt{\sum c}\cdot\sqrt{\sum(p-c)} = \frac{1}{r}\cdot\sqrt{2p}\cdot\sqrt{p} = \frac{p\sqrt{2}}{r}$$

Equality holds for $a=b=c$.