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In $\triangle ABC$ the following relationship holds:

$$\sum bc \cos^2 \frac{A}{2} \geq 3 \sum bc \sin^2 \frac{A}{2}$$

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$$\sum bc \cos^2 \frac{A}{2} = \sum bc \frac{s(s-a)}{bc} = s \sum (s-a) = s^2 \quad (1)$$

$$\begin{aligned} 3 \sum bc \sin^2 \frac{A}{2} &= 3 \sum bc \frac{(s-b)(s-c)}{bc} = \\ &= 3 \sum (s-b)(s-c) \stackrel{\forall x,y,z>0}{\leq} \frac{3 \sum xy \leq (\sum x)^2}{3} \left(\sum (s-a) \right)^2 = s^2 \quad (2) \end{aligned}$$

From (1)&(2) we get $\sum bc \cos^2 \frac{A}{2} \geq 3 \sum bc \sin^2 \frac{A}{2}$

Equality holds for $A = B = C$.