ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$\frac{a}{(b+c)^3}+\frac{b}{(c+a)^3}+\frac{c}{(a+b)^3}\ge\frac{27}{32s^2}$$

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Solution by Tapas Das-India

$$WLOG\ a\ge b\ge c\ then\ a+b\ge a+c\ge b+c$$

$$\frac{a}{(b+c)^3}+\frac{b}{(c+a)^3}+\frac{c}{(a+b)^3}=\sum\frac{a}{(b+c)^3}=$$

$$=\sum\left(\frac{a}{b+c}\cdot\frac{1}{(b+c)^2}\right)\overset{Chebyshev}{\ge}\frac{1}{3}\sum\frac{a}{b+c}\sum\frac{1}{(b+c)^2}=$$

$$=\frac{1}{3}\sum\frac{a}{b+c}\sum\frac{1^3}{(b+c)^2}\overset{Radon\ \&Nesbitt}{\ge}\frac{1}{3}\cdot\frac{3}{2}\cdot\frac{(1+1+1)^3}{(4s)^2}=\frac{27}{32s^2}$$

Equality holds for $a=b=c$.