ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$\frac{r_a^3}{(b+c)^2}+\frac{r_b^3}{(c+a)^2}+\frac{r_c^3}{(a+b)^2}\ge\frac{27r}{16}$$

Proposed by Mehmet Șahin-Turkiye

Solution by Tapas Das-India

$$\frac{(4R+r)^2}{s^2}\ge 3\ (1)\ and\ (4R+r)\overset{Euler}{\ge}9r\ \ (2)$$

$$\frac{r_a^3}{(b+c)^2}+\frac{r_b^3}{(c+a)^2}+\frac{r_c^3}{(a+b)^2}\overset{Radon}{\ge}\frac{(\sum r_a)^3}{(4s)^2}=\frac{(4R+r)^3}{16s^2}=$$

$$=\frac{(4R+r)^2}{s^2}\cdot\frac{4R+r}{16}\overset{(1)\&(2)}{\ge}\frac{27r}{16}$$

Equality holds for $a=b=c$.