ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$\frac{r_a}{b+c-r}+\frac{r_b}{c+a-r}+\frac{r_c}{a+b-r}>\frac{27r}{4s-3r}$$

Proposed by Mehmet Şahin-Turkiye

Solution by Tapas Das-India

$$WLOG\ a\ge b\ge c\ \ then\ r_a\ge r_b\ge r_c\ and\ a+b\ge a+c\ge b+c$$

$$\frac{r_a}{b+c-r}+\frac{r_b}{c+a-r}+\frac{r_c}{a+b-r}\ge$$

$$\overset{Chebyshev}{\ge}\frac{1}{3}\left(\sum r_a\right)\left(\sum\frac{1}{b+c-r}\right)\overset{Bergstrom}{>}$$

$$>\frac{1}{3}(4R+r)\cdot\frac{(1+1+1)^2}{4s-3r}\overset{Euler}{\ge}\frac{1}{3}\cdot 9r\cdot\frac{9}{4s-3r}=\frac{27r}{4s-3r}$$