ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$F \leq \frac{\sqrt{a^2b^2+b^2c^2+c^2a^2}}{4}$$

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Solution by Tapas Das-India

$$Heron's\ formula: 2\left(\sum a^2b^2\right)-\sum a^4 = 16F^2 \quad (1)$$

$$(b^2-c^2)^2+(c^2-a^2)^2+(a^2-b^2)^2 \geq 0\ or$$

$$b^2c^2+c^2a^2+a^2b^2 \leq a^4+b^4+c^4\ (2)$$

$$From\ (1): 16F^2 = 2\left(\sum a^2b^2\right)-\sum a^4 \overset{(2)}{\leq}$$

$$\leq\ 2\left(\sum a^2b^2\right)-\left(\sum a^2b^2\right)=\left(\sum a^2b^2\right)\ or$$

$$F \leq \frac{\sqrt{a^2b^2+b^2c^2+c^2a^2}}{4}$$

$$Equality\ for\ a=b=c$$