ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$\sqrt[6]{\prod_{cyc}(c+a-b)}\cdot\sqrt{\frac{a+b+c}{3}}\le\sqrt[3]{abc}$$

Proposed by Pavlos Trifon-Greece

Solution by Tapas Das-India

$$\sqrt[6]{\prod_{cyc}(c+a-b)}\cdot\sqrt{\frac{a+b+c}{3}}\le\sqrt[3]{abc}$$

$$\sqrt[6]{\prod_{cyc}(2s-2b)}\cdot\sqrt{\frac{2s}{3}}\le\sqrt[3]{4Rrs}$$

$$8(s-a)(s-b)(s-c)\cdot\frac{8s^3}{27}\le 16R^2r^2s^2$$

$$\frac{1}{s}\cdot s(s-a)(s-b)(s-c)\cdot\frac{8s^3}{27}\le 2R^2r^2s^2$$

$$\frac{F^2}{s}\cdot 4s\le 27R^2r^2$$

$$4r^2s^2\le 27R^2r^2$$

$$s^2\le\frac{27R^2}{4}$$

$$s\le\frac{3\sqrt{3}}{2}\cdot R\ (MITRINOVIC)$$

Equality holds for $a=b=c$.