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In any acute triangle ABC with semiperimeter p holds

$$a\sqrt{\cos A}+b\sqrt{\cos B}+c\sqrt{\cos C}\leq p\sqrt{2}$$

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By CBS inequality, we have

$$a\sqrt{\cos A}+b\sqrt{\cos B}+c\sqrt{\cos C}\leq\sqrt{(a+b+c)(a\cos A+b\cos B+c\cos C)},$$

and since $a+b+c=2p$ **and** $a\cos A+b\cos B+c\cos C=\frac{2F}{R}=p.\frac{2r}{R}\overset{Euler}{\leq}p$, **then**

$$a\sqrt{\cos A}+b\sqrt{\cos B}+c\sqrt{\cos C}\leq p\sqrt{2},$$

Equality holds iff ΔABC **is equilateral.**