ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$\sum_{cyc} \frac{a}{b\left(\cos^2\frac{A}{2}+\cos^2\frac{B}{2}\right)+c\cos^2\frac{C}{2}} \geq \frac{4}{3}$$

Proposed by Zaza Mzhavanadze-Georgia

Solution by Tapas Das-India

$$\sum \cos^2\frac{A}{2} = 2+\frac{r}{2R} \overset{Euler}{\leq} \frac{9}{4}(1)$$

$$\sum \frac{a}{b\left(\cos^2\frac{A}{2}+\cos^2\frac{B}{2}\right)+c\cos^2\frac{C}{2}} = \sum \frac{a^2}{ba\left(\cos^2\frac{A}{2}+\cos^2\frac{B}{2}\right)+ca\cos^2\frac{C}{2}} \overset{Bergstrom}{\geq}$$

$$\geq \frac{(\sum a)^2}{(\sum ab)\left(\sum \cos^2\frac{A}{2}\right)} \overset{(1)}{\geq} \frac{(\sum a)^2}{\frac{(\sum a)^2}{3}\frac{9}{4}} = \frac{4}{3}$$

$$Equality\ for\ a=b=c$$