In ΔABC the following relationship holds:

$$\frac{n_a^4}{sinA}+\frac{n_b^4}{sinB}+\frac{n_c^4}{sinC}\ge 162\sqrt{3}r^4$$

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Solution by Daniel Sitaru-Romania

$$\frac{n_a^4}{sinA}+\frac{n_b^4}{sinB}+\frac{n_c^4}{sinC}=\sum_{cyc}\frac{n_a^4}{sinA}\ge\sum_{cyc}\frac{h_a^4}{sinA}=$$

$$=\sum_{cyc}\frac{\left(\frac{2F}{a}\right)^4}{sinA}=16F^4\sum_{cyc}\frac{1}{a^4 sinA}=16F^4\sum_{cyc}\frac{1}{a^4\cdot\frac{a}{2R}}=$$

$$=32F^4R\sum_{cyc}\frac{1}{a^5}=32r^4s^4R\sum_{cyc}\frac{1^6}{a^5}\overset{RADON}{\ge}$$

$$\ge 32r^4s^4R\cdot\frac{(1+1+1)^6}{(a+b+c)^5}=\frac{32r^4s^4R\cdot 3^6}{32s^5}=$$

$$=\frac{3^6r^4R}{s}\overset{MITRINOVIC}{\ge}\frac{3^6r^4R}{\frac{3\sqrt{3}}{2}R}=\frac{2\cdot 243}{\sqrt{3}}r^4=162\sqrt{3}r^4$$

Equality holds for: $a=b=c$.