ROMANIAN MATHEMATICAL MAGAZINE

If $n \in \mathbb{N}$ then in ΔABC the following relationship holds:

$$\frac{a^n}{\sin A}+\frac{b^n}{\sin B}+\frac{c^n}{\sin C} \ge \left(2\sqrt{3}\right)^{n+1}\cdot r^n$$

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Solution by Daniel Sitaru-Romania

$$\frac{a^n}{\sin A}+\frac{b^n}{\sin B}+\frac{c^n}{\sin C}=\sum_{cyc}\frac{a^n}{\sin A}=$$

$$=\sum_{cyc}\frac{a^n}{\frac{a}{2R}}=2R\cdot\sum_{cyc}a^{n-1}=2R\cdot\sum_{cyc}\frac{a^{n-1}}{1^{n-2}}\overset{RADON}{\ge}$$

$$\ge 2R\cdot\frac{(a+b+c)^{n-1}}{(1+1+1)^{n-2}}=2R\cdot\frac{(2s)^{n-1}}{(3)^{n-2}}\overset{MITRINOVIC}{\ge}$$

$$\ge 2R\cdot\frac{2^{n-1}\cdot\left(3\sqrt{3}r\right)^{n-1}}{3^{n-2}}\overset{EULER}{\ge}2\cdot 2r\cdot 2^{n-1}\cdot 3^{\frac{3n-3}{2}-n+2}\cdot r^{n-1}=$$

$$=2^{n+1}\cdot 3^{\frac{n+1}{2}}\cdot r^n=\left(2\sqrt{3}\right)^{n+1}\cdot r^n$$

Equality holds for $a=b=c$.