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In ΔABC the following relationship holds:

$$\frac{a^n}{\sin\frac{A}{2}\sin\frac{B}{2}}+\frac{b^n}{\sin\frac{B}{2}\sin\frac{C}{2}}+\frac{c^n}{\sin\frac{C}{2}\sin\frac{A}{2}}\geq 2^{n+2}.3^{\frac{(n+2)}{2}}.r^n,\qquad \forall n\in N$$

Proposed by Zaza Mzhavanadze-Georgia

Solution by Tapas Das-India

$$abc=4Rrs\overset{Euler\ \&\ Mitrinovic}{\geq}4.2r.r.3\sqrt{3}r=\left(2r\sqrt{3}\right)^3\ (1)$$

$$and\ \prod\sin^2\frac{A}{2}=\left(\frac{r}{4R}\right)^2\overset{Euler}{\leq}\frac{1}{64}\quad(2)$$

$$\frac{a^n}{\sin\frac{A}{2}\sin\frac{B}{2}}+\frac{b^n}{\sin\frac{B}{2}\sin\frac{C}{2}}+\frac{c^n}{\sin\frac{C}{2}\sin\frac{A}{2}}\overset{Am-Gm}{\geq}\frac{3(abc)^{\frac{n}{3}}}{\left(\prod\sin^2\frac{A}{2}\right)^{\frac{1}{3}}}\overset{(1)\&(2)}{\geq}$$

$$\geq\ 3\cdot 2^n\cdot r^n\cdot 3^{\frac{n}{2}}\cdot 4=2^{n+2}\cdot 3^{\frac{(n+2)}{2}}\cdot r^n$$

$$Equality\ for\ a=b=c$$